Name: ______________________________________ Date: __________

Skill and Practice

4J Rate of Change of Momentum

Read:

Momentum is given by the expression $p = mv$ where p is the momentum of an object of mass m moving with velocity v. The units of momentum are *kg-m/s*. Change of momentum (represented Δp) over a time interval (represented Δt) is also called the rate of change of momentum.

Since, momentum is $p = mv$, if the mass remains constant during the time Δt, then:

$$\frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t}$$

The expression, $\frac{\Delta v}{\Delta t}$, represents change in velocity over change in time, also known as acceleration. From Newton's second law, we know that acceleration equals force divided by mass ($a = F/m$). Rearranging the equation, we see that force equals mass times acceleration ($F = ma$). Similarly, force (F) equals change in momentum over change in time.

$$F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$$

A mass, m, moving with velocity, v, has momentum mv. If this momentum becomes zero over some change in time (Δt), then there is a force, $F = (mv - 0)/\Delta t$.

- mv is the initial momentum.
- 0 is the momentum after a change in time Δt.

When a car accelerates or decelerates, we feel a force that pushes back during acceleration and pushes us forward during deceleration. When the car brakes slowly, the force is small. However, when the car brakes quickly, the force increases considerably.

Example:

Example 1. An 80-kg woman is a passenger in a car going 90 km/h. The driver puts on the brakes and the car comes to a stop in 2 seconds. What is the average force felt by the passenger?

First, convert the velocity to a value that is in meters per second: 90 km/h = 25 m/s. Next, use the equation that relates force and momentum:

$$\text{Force} = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = 80\ \text{kg}\frac{(25 - 0)\ \text{m/s}}{2\ \text{s}} = 1{,}000\ \text{N}$$

This is a large force, and for the passenger to stay in her seat, she must be strapped in with a seat belt.

When the stopping time decreases from 2 seconds to 1 second, the force increases to 2,000 newtons. When the car is involved in a crash, the change in momentum happens over a much shorter period of time, thereby creating very large forces on the passenger. Air bags and seat belts help by slowing down the person's momentum change, resulting in smaller forces and a reduced chance for injury. Let's look at some numbers.

The car travels at 90 kilometers per hour, crashes, and comes to a stop in 0.1 seconds. The air bag inflates and cushions the person for 1.5 seconds. Let's calculate the force experienced by the passenger in an automobile without air bags and in one case with air bags.

- Without the air bag, the momentum change happens over 0.1 seconds. This results in a force:

$$\text{Force} = 80\ \text{kg}\frac{25\ \text{m/s}}{0.1\ \text{s}} = 2{,}000\ \text{N}$$

  The human body is not likely to survive a force as large as this.

- With the air bag, the force created is:

$$\text{Force} = 80\ \text{kg}\frac{25\ \text{m/s}}{1.5\ \text{s}} = 1{,}333\ \text{N}$$

  The chances for survival are much higher.

Example 2. A pile is driven into the ground by hitting it repeatedly. If the pile is hit by the driver mass at a rate of 100 kg/s and with a speed of 10 m/s, calculate the resulting average force on the pile.

We are told that the driver mass hits the pile at a rate of 100 kg/s. What does this mean exactly? We can have a 100-kilogram mass hitting the pile every second, or a 50-kilogram mass hitting the pile every half-second, or a 200-kilogram mass hitting the pile every 2 seconds. You get the idea.

The speed (v) with which the mass hits the pile is 10 m/s. The mass (m) is 100 kilograms. Time changes occur at 1-second intervals. The force on the pile is:

$$\text{Force} = m\frac{\Delta v}{\Delta t} = 100\ \frac{\text{kg}}{\text{s}}\ 10\ \frac{\text{m}}{\text{s}} = 1{,}000\ \frac{\text{kg m}}{\text{s}^2} = 1{,}000\ \text{N}$$

Practice:

1. A 1,000-kg wrecking ball hits a wall with a speed of 2 m/s and comes to a stop in 0.01 s. Calculate the force experienced by the wall.

---

2. A 0.15-kg soccer ball is rolling with a speed of 10 m/s and is stopped by the frictional force between it and the grass. If the average friction force is 0.5 N, how long would this take?

---

3. Water comes out of a fire hose at a rate of 5.0 kg/s and with a speed of 50 m/s. Calculate the force on the hose. (This is the force that the firefighter has to provide in order to hold the hose.)

4. Water from a fire hose is hitting a wall straight on. The water comes out with a flow rate of 25 kg/s and hits the wall with a speed of 30. m/s. What is the resulting force exerted on the wall by the water?

5. The water at Niagara Falls flows at a rate of 3.0 million kg/s. The water hits the bottom of the falls at a speed of 25 m/s. What is the force generated by the change in momentum of the falling water?

6. A 50-g (0.050-kg) egg that is dropped from a height of 5.0 m will hit the floor with a speed of about 10. m/s. The hard floor forces the egg to stop very quickly. Let's say that it will stop in 0.0010 second.
   a. What is the force created on the egg?

   b. The egg will break at the force you calculated for 6(a). Imagine that a 50-kilogram person fell down on the egg falling under the influence of gravity. What would the force of the person on the egg be?

   c. Do you think the egg will break if the person fell on it? Why or why not?

   d. If we now drop the egg onto a pillow, it will allow the egg to stop over a much longer time compared with the time it takes for it to stop on the hard surface. The weight and the velocity of the egg is still the same, but now the time it takes for the egg to come to rest is much longer, about 0.5 second or about 500 times longer than the time it took to stop on the floor. What would the force on the egg be under these circumstances?

   e. Do you think the egg will break when it drops on the pillow? Why or why not?